UNCLASSIFIED

TECHNICAL NOTE
PAD 94

# DEPARTMENT OF SUPPLY

## AUSTRALIAN DEFENCE SCIENTIFIC SERVICE

## WEAPONS RESEARCH ESTABLISHMENT

SALISBURY, SOUTH AUSTRALIA

TECHNICAL NOTE PAD 94

# A CIRCULAR TO RECTANGULAR WAVEGUIDE TRANSITION MAINTAINING A CONSTANT CUTOFF WAVELENGTH

J.R. PYLE

COPY No. 42

SEPTEMBER 1964

UNCLASSIFIED

# Report Documentation Page

*Form Approved*
*OMB No. 0704-0188*



| | | |
|---|---|---|
| 1. REPORT DATE<br>**JAN 2008** | 2. REPORT TYPE | 3. DATES COVERED<br>**00-00-2008 to 00-00-2008** |

| | |
|---|---|
| 4. TITLE AND SUBTITLE<br>**A Circular to Rectangular Waveguide Transition Maintaining a Constant Cutoff Wavelength** | 5a. CONTRACT NUMBER<br>5b. GRANT NUMBER<br>5c. PROGRAM ELEMENT NUMBER |
| 6. AUTHOR(S) | 5d. PROJECT NUMBER<br>5e. TASK NUMBER<br>5f. WORK UNIT NUMBER |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>**Department of Supply,Australian Defence Scientific Service,Weapons Research Establishment,Salisbury, South Australia,** | 8. PERFORMING ORGANIZATION REPORT NUMBER |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) | 10. SPONSOR/MONITOR'S ACRONYM(S)<br>11. SPONSOR/MONITOR'S REPORT NUMBER(S) |

12. DISTRIBUTION/AVAILABILITY STATEMENT
**Approved for public release; distribution unlimited**

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

| 16. SECURITY CLASSIFICATION OF: | | | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON |
|---|---|---|---|---|---|
| a. REPORT<br>**unclassified** | b. ABSTRACT<br>**unclassified** | c. THIS PAGE<br>**unclassified** | **Same as Report (SAR)** | **15** | |

UNCLASSIFIED

DEPARTMENT OF SUPPLY

AUSTRALIAN DEFENCE SCIENTIFIC SERVICE

WEAPONS RESEARCH ESTABLISHMENT

TECHNICAL NOTE PAD 94

# A CIRCULAR TO RECTANGULAR WAVEGUIDE TRANSITION MAINTAINING A CONSTANT CUTOFF WAVELENGTH

J.R. Pyle




## S U M M A R Y

A circular to rectangular waveguide transition is described. The transition maintains a constant cutoff wavelength equal to that of the fundamental wave ($TE_{11}$) in circular waveguide.

The parameters of the variable cross-section throughout the transition are computed and normalised to the radius of the circular waveguide.


POSTAL ADDRESS: The Director, Weapons Research Establishment, Box 1424H, G.P.O., Adelaide, South Australia.

UNCLASSIFIED

## TABLE OF CONTENTS



## LIST OF FIGURES

## 1. INTRODUCTION

This communication describes a circular to rectangular waveguide transition which maintains a constant cutoff wavelength, and hence a constant phase velocity, of the fundamental wave ($TE_{11}$ or $TE_{10}$). The constant cutoff wavelength is that of the $TE_{11}$ wave in the circular waveguide.

## 2. THEORY

An earlier communication(1) described the computation of the phase shift introduced by inserting a pair of diametrically opposite metal plates normal or parallel to the electric field, E of the $TE_{11}$ wave in a circular waveguide of radius r. Figure 1 illustrates the phase shift effects introduced by the plates of penetration depths $d_1$ and $d_2$. Since the effects are of opposite sign for the cases when the plates are normal or parallel to the electric field it is possible, with correctly dimensioned plates, to introduce both pairs of plates to maintain a constant phase shift i.e. a constant cutoff wavelength.

Figure 2 shows the limiting case of a circular waveguide transformed to a rectangular waveguide of dimensions a and b by means of two pairs of plates of penetration depths $d_{1MAX}$ and $d_{2MAX}$. The values of $d_{1MAX}$ and $d_{2MAX}$ are readily computed as follows:

The cutoff wavelength of the $TE_{11}$ wave in the circular waveguide is(2)

$$\lambda_C\,(TE_{11}) = 2\pi r / 1.841184 \qquad (1)$$

The cutoff wavelength of the $TE_{10}$ wave in the rectangular waveguide is(2)

$$\begin{aligned}\lambda_c\,(TE_{10}) &= 2a \qquad (2)\\ &= 2\,(2r - 2d_{2MAX})\end{aligned}$$

Equating $\lambda_C\,(TE_{10})$ and $\lambda_C\,(TE_{11})$ and solving for $d_{2MAX}$ we obtain

$$\begin{aligned}\frac{d_{2MAX}}{r} &= 1 - \frac{\pi}{2 \times 1.841184}\\ &= 0.146875\end{aligned}$$

The value of $d_{1MAX}$ follows from the geometry of the system

$$\begin{aligned}\frac{d_{1MAX}}{r} &= 1 - \sqrt{\left[1 - \left(1 - \frac{d_{2MAX}}{r}\right)^2\right]} \qquad (3)\\ &= 0.47830\end{aligned}$$

Figure 3 shows a typical cross-section of the transition and the expected curve of $\frac{d_2}{r}$ against $\frac{d_1}{r}$ for a constant $\lambda_c$ ($TE_{11}$).

The computation of the values of $\frac{d_1}{r}$ and $\frac{d_2}{r}$ is now described.

## 3. COMPUTATION OF PLATE DEPTHS

An earlier communication(3) describes the computation of the cutoff wavelength of waveguides with perturbed cross-sections in either the electric (E) or magnetic (H) plane. The present analysis differs in that both E and H plane perturbations exist and the cutoff wavelength is fixed.

Figure 4 shows one quadrant of the cross-section of a circular waveguide of radius r divided into N rectangular strips of equal width, $\frac{r}{N}$ and variable height, $h_g$. The cutoff condition of the $TE_{11}$ wave may be determined by considering the fundamental TEM wave travelling normal to the axis of the waveguide. Each rectangular strip is regarded as an elementary transmission line and may be represented by the transmission matrix(4).

$$\left[ Z_g \right] = \begin{bmatrix} \cos\theta & j\,Z_g \sin\theta \\ \dfrac{j \sin\theta}{Z_g} & \cos\theta \end{bmatrix} \qquad (5)$$

The normalised impedance, $Z_g$, of the general $g^{th}$ strip with respect to the first is given by

$$Z_g = \frac{h_g}{h_1}$$

The electrical length, $\theta$ of each elementary transmission line is given by

$$\theta = \frac{2\pi r}{N \lambda_c} \qquad (6)$$

where $\lambda_c$ is obtained from (1).

The model used to compute the values of $d_1$ and $d_2$ to maintain a constant cutoff wavelength is shown in figure 5. The value of $d_2$ is set equal to an integral number, q of rectangular strips i.e. a short circuit exists at C at the end of the (N-q)th elementary transmission line. The object is to determine the number of strips, p each of which have the same height, $h_p$ as the p th strip and which transform the short circuit at C to an open circuit at AB. The value of $d_1$ is given by $r-h_p$.

The voltage V, and current I at AB (figure 5), are given by the matrix equation.

$$\begin{bmatrix} V \\ \hline I \end{bmatrix} = \begin{bmatrix} Z_p \end{bmatrix}^P \times \begin{bmatrix} Z_{p+1} \end{bmatrix} \times \ldots \begin{bmatrix} Z_{N-q} \end{bmatrix} \times \begin{bmatrix} 0 \\ \hline 1 \end{bmatrix} \qquad (7)$$

which reduces to

$$\begin{bmatrix} V \\ \hline I \end{bmatrix} = \begin{bmatrix} f(\theta) \\ \hline g(\theta) \end{bmatrix} \qquad (8)$$

where $f(\theta)$ and $g(\theta)$ are functions of $\theta$.

The $TE_{11}$ wave in the circular waveguide is cut off when the impedance at AB (figure 5) is infinite i.e. when $g(\theta) = 0$.

A digital computer may be programmed to search for a value of p in(7) such that $g(\theta)$ is zero. The depth, $d_1$ of the E plane plate is determined from

$$d_1 = r - h_p \qquad (9)$$

## 4. ERROR ANALYSIS

In the above determination no account has been taken of the curvature of the electric field, E across the waveguide. The effect of considering only the fundamental TEM wave is to introduce a bias error(3), which is approximately constant at 2.41 per cent for large N and small perturbations in either the E or H plane. When the circular waveguide is transformed to a rectangular waveguide (figure 2) it is quite correct to consider only the fundamental TEM wave and no bias error correction is required. To compute the curve of $\frac{d_2}{r}$ against $\frac{d_1}{r}$ the bias error must be weighted to allow for the extent of perturbation. The correct weighting factor is found by trial and error and figure 6 shows some computed curves using no bias error correction, a bias error correction with cosine weighting and a bias error correction with $1 - (\text{sine})^4$ weighting. It was decided to use the $1 - (\text{sine})^4$ weighting as this caused the curve to pass through the origin. Since the bias error is so small it is quite permissible to choose a weighting factor by satisfying the end conditions.

## 5. COMPUTED RESULTS AND CONCLUSION

The computed values of $\frac{d_1}{r}$ and $\frac{d_2}{r}$, which may be used to design a circular to rectangular waveguide transition with a constant cutoff wavelength, are shown in figure 7. The results are given in both graphical and tabular forms for the convenience of the microwave designer.

## 6. ACKNOWLEDGEMENTS

The author wishes to thank Mr. F.J. Lehany, Chief of the Division of Applied Physics, C.S.I.R.O. for suggesting this problem as an extension to earlier work(3).

## REFERENCES


| No. | Author | Title |
|---|---|---|
| 1 | Pyle, J.R. | "Circular Polarisers of Fixed Bandwidth". IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-12, September 1964. |
| 2 | Schelkunoff, S.A. | "Electromagnetic Waves". D.Van Nostrand Co., Inc., New York, N.Y., 1st. Ed., Chapter 8, 1943. |
| 3 | Pyle, J.R. and Angley, R.J. | "Cutoff Wavelengths of Waveguides with Unusual Cross-Sections". IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-12, September 1964. |
| 4 | Ghose, R.N. | "Microwave Circuit Theory and Analysis". McGraw-Hill Book Co., Inc., New York, N.Y. 1st. Ed. Ch. 10, 1963. |

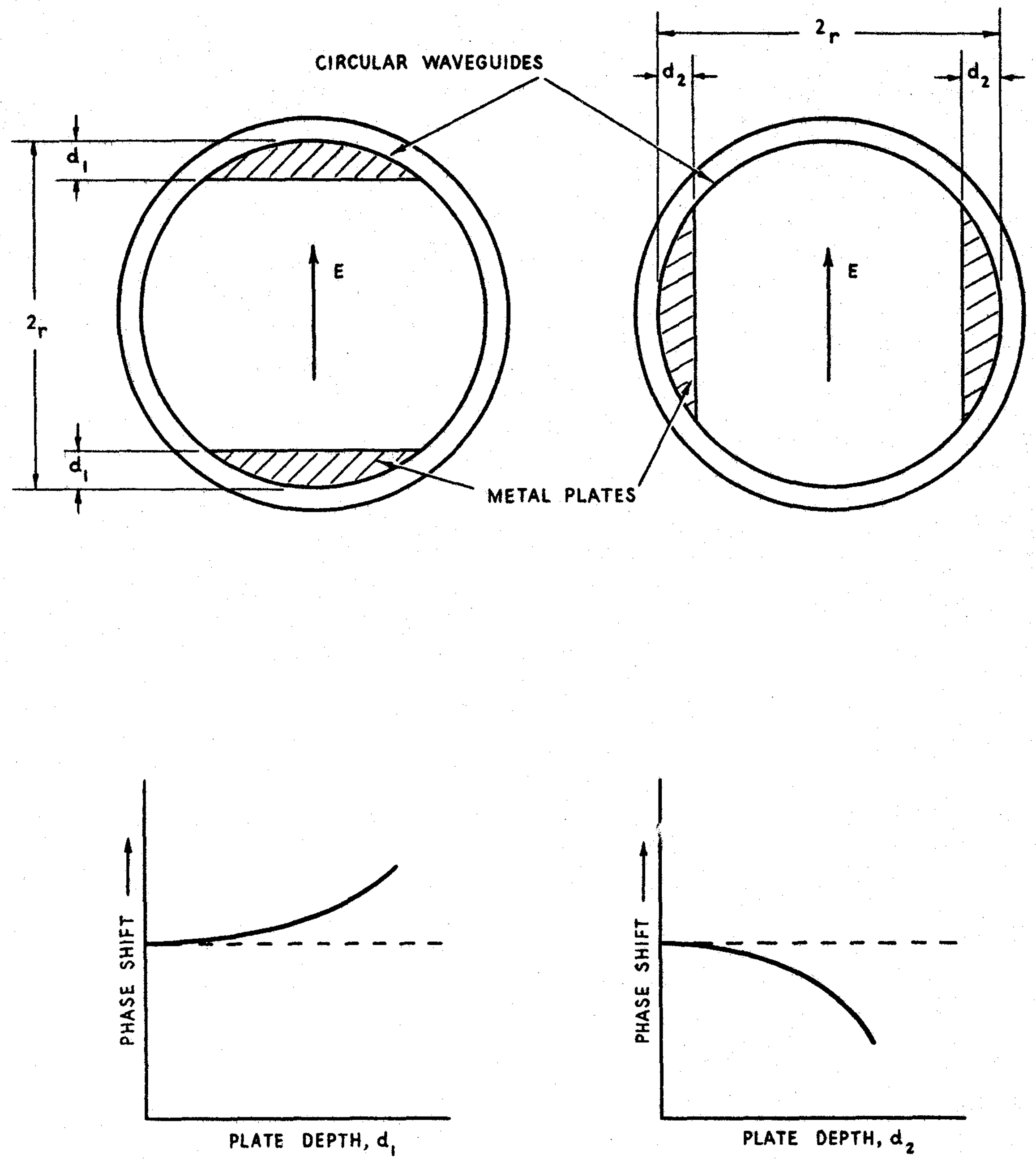



FIGURE 1. PHASE SHIFT EFFECT OF METAL PLATES IN CIRCULAR WAVEGUIDE

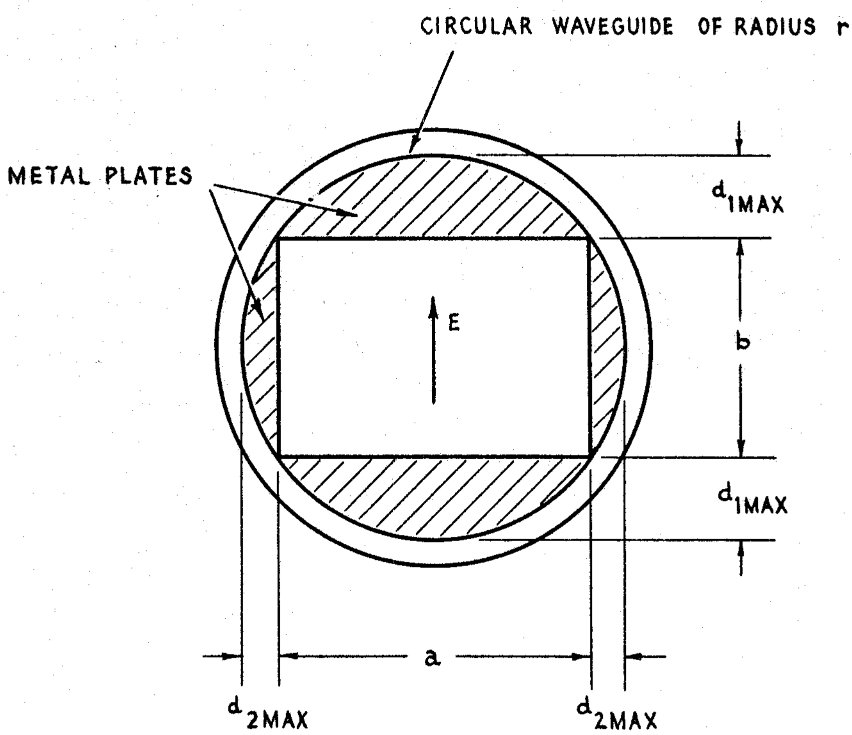
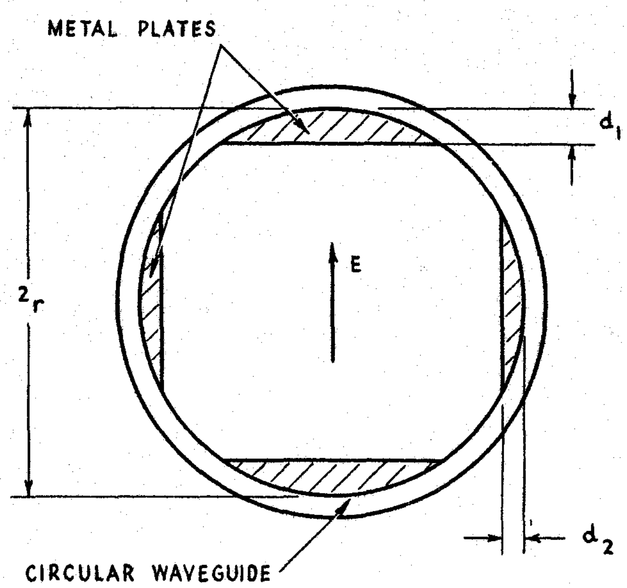



FIGURE 2. CIRCULAR WAVEGUIDE TRANSFORMED TO RECTANGULAR WAVEGUIDE WITH METAL PLATES



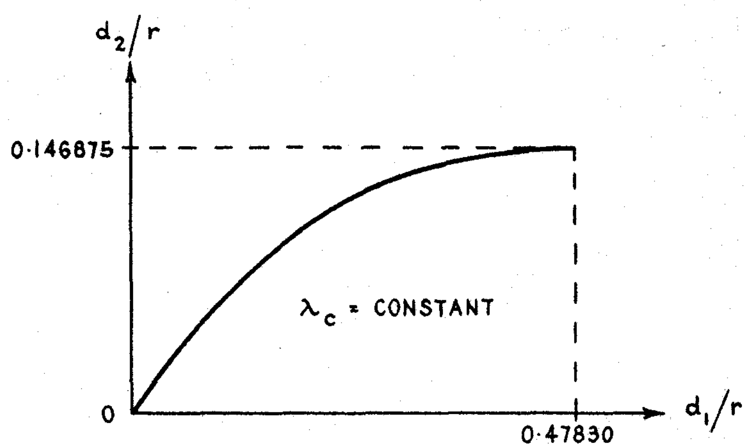




FIGURE 3. TYPICAL CROSS-SECTION OF TRANSITION AND EXPECTED CURVE FOR $d_2/r$ PLOTTED AGAINST $d_1/r$ FOR CONSTANT $\lambda_C$

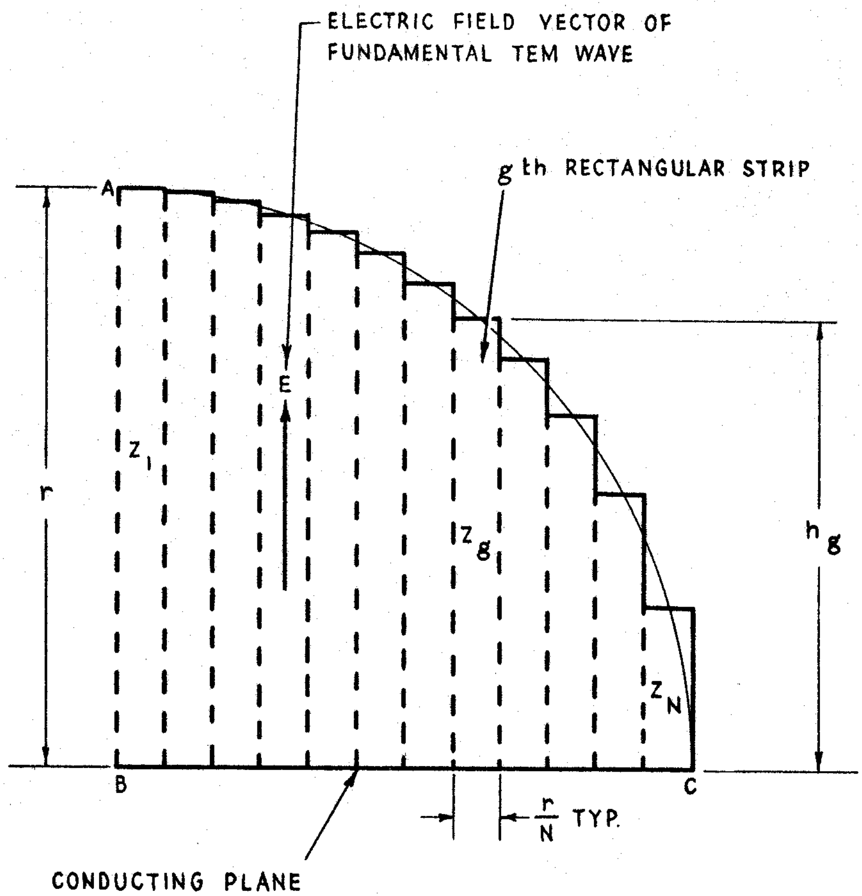



FIGURE 4. QUADRANT OF CIRCULAR WAVEGUIDE CROSS-SECTION REPRESENTED BY N RECTANGULAR STRIPS

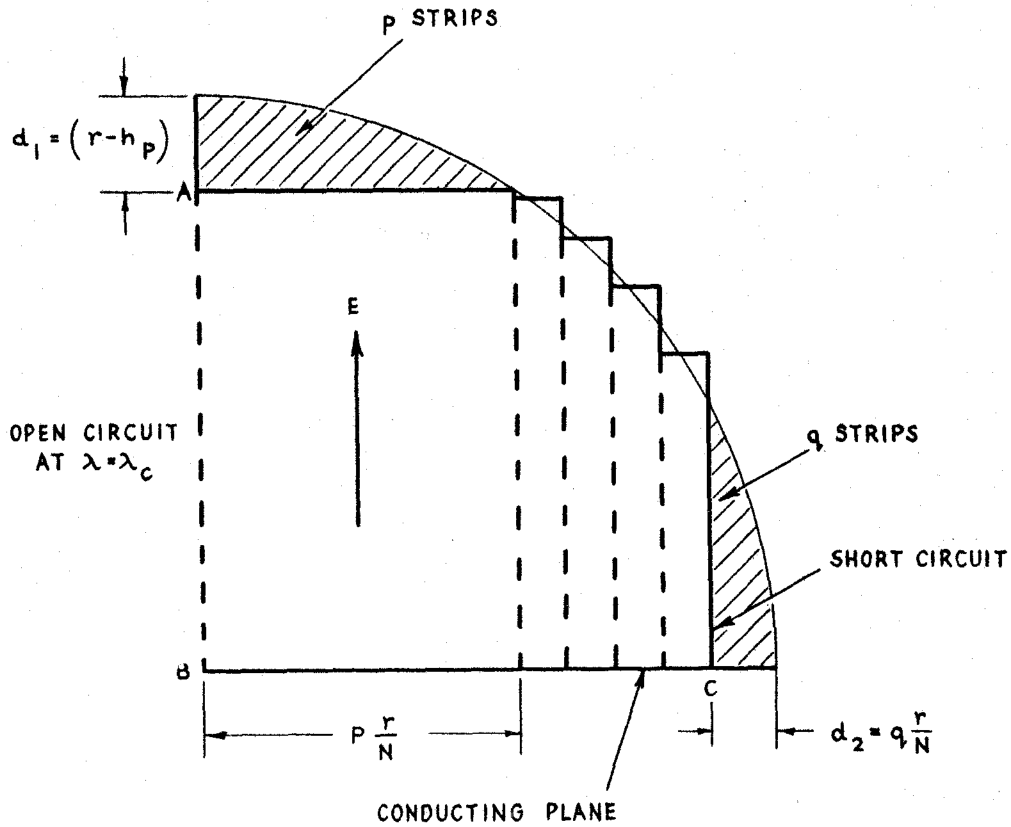



FIGURE 5. MODEL FOR COMPUTATION OF E AND H PLANE PLATE DEPTHS FOR CONSTANT $\lambda_C$

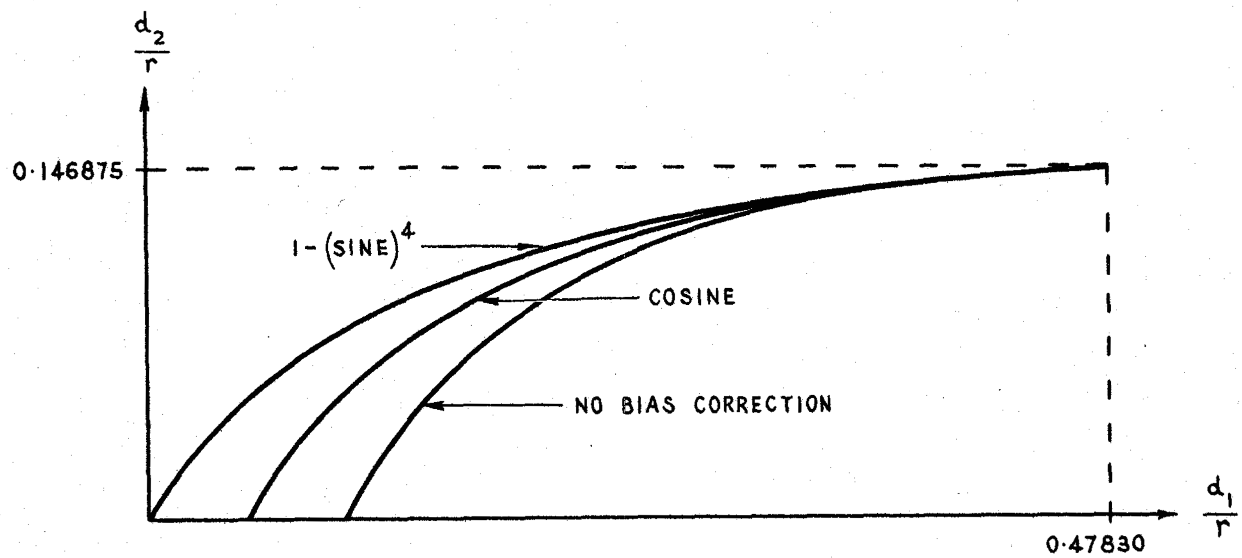
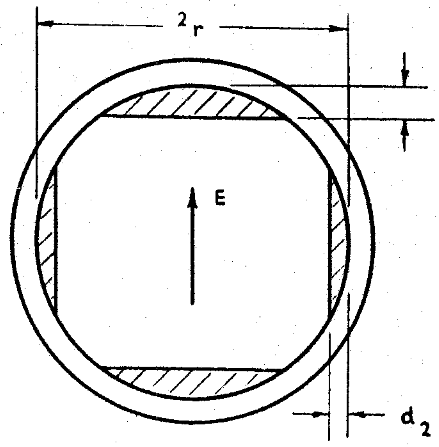
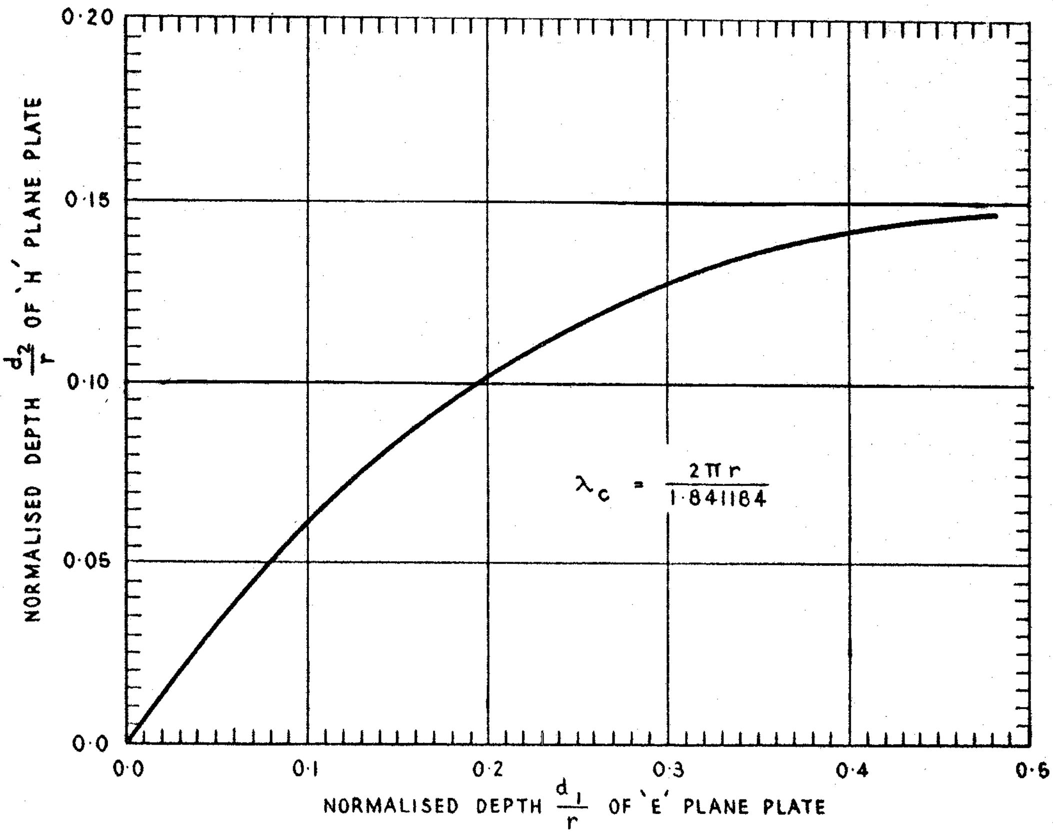



FIGURE 6. EFFECT OF DIFFERENT WEIGHTING FACTORS FOR BIAS ERROR IN COMPUTING VALUES OF $d_1/r$ AND $d_2/r$




| COMPUTED VALUES | | | |
|---|---|---|---|
| $d_2/r$ | $d_1/r$ | $d_2/r$ | $d_1/r$ |
| 0·0100 | 0·0148 | 0·0900 | 0·1648 |
| 0·0200 | 0·0288 | 0·1000 | 0·1946 |
| 0·0300 | 0·0430 | 0·1100 | 0·2280 |
| 0·0400 | 0·0580 | 0·1200 | 0·2660 |
| 0·0500 | 0·0746 | 0·1300 | 0·3111 |
| 0·0600 | 0·0934 | 0·1400 | 0·3715 |
| 0·0700 | 0·1145 | 0·1450 | 0·4233 |
| 0·0800 | 0·1381 | 0·1469 | 0·4783 |

FIGURE 7. COMPUTED VALUES OF $d_1/r$ AND $d_2/r$ TO GIVE A CIRCULAR TO RECTANGULAR WAVEGUIDE TRANSITION WITH A CONSTANT CUTOFF WAVELENGTH